Geoffrey William Sweet,
The Institute for Advanced Studies in the Humanities,
The University of Edinburgh,
Hope Park Square,
Edinburgh
EH8 9NW

History and the principle of uncertainty

My purpose is to extend a mathematical principle of uncertainty to the historical sciences, and thus explain the connections between Enlightenment Statistics, which I shall define in due course, and *Historismus*, a term which crystallizes the dominant historical outlook and method in the nineteenth and early twentieth centuries in Germany. Friedrich Meinecke (1862-1954), perhaps the greatest exponent of *Historismus*, and to whom I shall return later, defined *Historismus* as 'the idea of inimitable, singular individuality, developing according to its very own organic laws of existence, an individuality that cannot be comprehended by means of logical thought alone, let alone of the mechanical law of causation'.[1] Historists claimed that eighteenth century historiography had had but a negligible influence upon *Historismus*, but they sometimes allowed that the elements of it were present in the works of writers such as Herder and Goethe who were not historians in the technical sense. Historists often argued that the approaches of the eighteenth century historians were the products of the general outlook of their age. Thus they depicted Enlightenment historiography as an endorsement of rationalism and cosmopolitanism. But they argued that they themselves had nourished as well as embraced Romanticism, nationalism and irrationalism. Likewise, they advertized *Historismus* partly as the catalyst and partly as a result of a general rejection of the Enlightenment and of a restoration or inception of political conservatism at the beginning of the nineteenth century.

I shall conjecture that neither Statistics nor *Historismus* were second-order effects, that is to say, derivatives of a general moral and political outlook, but that the key to the difference between them lies in their methods which were at variance with one another. Furthermore, I shall argue that there is a basic uncertainty involved in combining the two simultaneously.

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[1] 'Es ist vielmehr die Idee der Individualität, die zum Entwicklungsgedanken hinzukommen mußte, um den vorzugsweise in Deutschland blühenden Historismus hervorzubringen - die Idee der unnachahmlichen, eigenartigen, nach eigenstem organischen Lebensgesetze sich entwickelnden Individualität, die mit den Mitteln des logischen Denkens, geschweige denn des mechanischen Kausalgesetzes allein nicht zu begreifen ist.' Friedrich Meinecke, 'Ernst Troeltsch und das Problem des Historismus' (1823), in Meinecke, *Zur Theorie und Philosophie der Geschichte*, ed. Eberhard Kessel, Stuttgart, 1959, p. 373f.

I take Statistics to epitomize the historical outlook of the German Enlightenment. Its purpose was to isolate as many variables as possible in what was assumed to be the political, economic and social continuum. Historists, on the other hand, regarded politics as the primary motor of history and treated the social and economic as consequences of political actions and events. In contrast to Statistics, *Historismus* concerned itself with comparatively few variables, a restrictive strategy that was forced upon it by the essence of its method which was to assume that human institutions were predominantly explicit functions of time rather than of invariant human needs and aspirations. The principal cause of the demise of *Historismus* was that its selection of variables was superseded by events. After the First World War, for instance, Historists, wishing to find out why they had not predicted the outcome of the war correctly, noticed that the so-called 'ideas of 1914' which they had canvassed were scarcely different from the 'ideas of 1813', which their nineteenth-century predecessors had seen as the leitmotifs of Germany's striving for unity and prosperity. Historists had made a very thorough study of the variables which they thought mainly constituted the system of German political advance from the Wars of Liberation through the nineteenth century. However, the pertinent variables had changed in the intervening century, and I argue that such changes could only have been elicited by Statistical methods, methods however that were absent from and had indeed been rejected by *Historismus*. But, in any case, I argue that, in theory, the two approaches, self-consistent though they may be within themselves, cannot be combined simultaneously 'to arbitrary precision' in the words of the German quantum physicist Werner Heisenberg; and this state of affairs I characterize in terms of his 'uncertainty principle'. Unlike the Historists, the eighteenth century Statists tried to combine the two approaches, but they were unable to generate an effective synthesis. Nevertheless, the Statists' efforts to reconcile the two methodologies are noteworthy, especially because they are strongly analogous to ones being made recently in the field of mathematical optimization.

Quantum Physics is based on the experience that matter behaves as if it consisted of particles, whose energies have discrete values called quanta. This is the great discovery of physics at the turn of the century. The term 'uncertainty principle' acknowledges that it is impossible to estimate both the location and the momentum of a particle simultaneously to arbitrary precision. And yet one needs to know something about both these things. Momentum is clearly something to do with speed, and the direction is also involved. The obvious way of finding out how fast a particle is moving would be by timing its departure from one point in space and its arrival at another. But the particle may be accelerating, in which case more than two points of observation are required. Short of being certain in advance that the particle will reach an actual location, statistical methods involving likelihood or probability are used to predict the particle's trajectory. But even if the points through which the particle's trajectory pass can be determined, it is ultimately impossible to tell by what route it has reached them. It could have gone by any of an infinite number of possible routes, and it is impossible to monitor its progress at every point in space. Therefore it is theoretically impossible to determine the particle's velocity by means of timing. The object of the experiment would be to reduce the number of possible trajectories. The momentum, however, can be measured by other means, such as the effect of the impact on a target.

But there is a more fundamental and difficult problem still, and that is how matter is to be decomposed into particles in the first place. The fundamental particles of physics, the atom, molecule, neutron, electron and so on are no more than elements of a theoretical model applied to the actuality. There is no theoretical basis for proving that matter is really like that. The quantum physicist therefore has to make an ultimately arbitrary decision about the quantum or particle of interest. In practice, the existence and nature of particles is estimated by demonstrating that a certain effect occurs again and again under the same experimental conditions. Where that happens, an invariant is said to have been identified.

But it can never be certain that precisely the same conditions have been reproduced from one experiment to the next. It may happen, for example, that the results of the experiment have been influenced by the conduct or environment of the experiment itself. It cannot be certain that the quantum or particle of interest might have behaved differently if it had not been confined by the experiment. It is a corollary of Heisenberg's principle then that the very act of examining a particle's behaviour may cause the particle to behave differently from how it would have behaved if it had been left free.

Results in quantum physics are obtained by repeating experiments. These entail what are known as 'statistical' proofs, that is, proofs based upon likelihood or probability. Strictly speaking, they are not mathematical proofs in the classical sense, where it had been axiomatic to prove that a problem had a solution in the first place and that it would be unique. The difference between the classical and statistical kinds of proof lies in the relative importance attached to induction and deduction. In classical Mathematics induction predominates while in Statistics deduction predominates. Recently a further development of the classical requirement of existence and uniqueness theorems is found in 'heuristic' methods. 'Heuristic' signifies that the method applied has not been or could not be proved in terms of an existence and uniqueness theorem. The heuristic approach is taken in the hope that many experiments will yield data from which information will be construed. Deduction predominates in the heuristic approach even more than it does in the statistical one.

'Bayesian' is a term that crops up more and more today in the activity of reconciling induction and deduction. The term arises from a paper entitled 'An essay towards solving a problem in the doctrine of chances' attributed to Thomas Bayes and communicated to the Royal Society upon his death in 1763. The paper, edited and and completed by Price at the time, is a mathematical treatment of the prediction of future events. More technically, it shows that the algebra to be used entails the inversion of a probability. The paper contains an expression for the result of experiments conducted in the past and a consideration of the probability of the same result occurring in a future experiment. In the expression, the probabilistic variable appears as a denominator on the right hand side of an equation. Prediction of the value of it requires proving the feasibility of taking the reciprocal of the probability and rearranging the equation so that the probability appears as a result on the left hand side of the equation rather than as a condition on the right hand side. However, a probability must be treated as an uncertainty in Mathematics, and there is no theoretical basis for conceiving of its reciprocal. Although today's exponents of 'Bayesian' methods implicitly assume the existence of the reciprocals of such uncertainties, a reading of Bayes' unfinished paper reveals that Bayes himself was unable to prove it.

The rejection of the notion of invertible probability is axial to eighteenth century German Statistics. The Statists strove to this conclusion in much the same way as did their contemporary Bayes, namely by a process of falsification. They started out by assuming that future developments could be predicted on the evidence of past ones, but they found that the hypothesis could not be tested. Just as Bayes found that the notion of invertible probability could not be proved in algebra, so the Statists discovered that human institutions could not be proved to develop as a function of time. Instead, they attended to the invariants of human institutions.

In *Historismus* the quantum of history was the nation. Historists likened the nation state to a living organism that changed constantly over time. In practice, the nation proved its existence by prevailing over other possible nations. In mathematical terms, *Historismus* was a simple linear time series analysis of the magnitude of the quantum over time. A nation manifested itself as a wave, rising and falling. Hermann Hettner, recalling in mid-nineteenth century the famous quarrel about method of a century before between Herder and Schlözer, a leading Statist to whom I shall return most often in this article, asserted that Herder's 'dynamic' view of history had been far superior to Schlözer's, but acknowledged that both thinkers had aimed to 'raise the study of history above the tedious clutter to the history of a humanity now striding forward, now degenerating.'[2] However, that concentration on the scale of linear growth made it difficult to examine the interaction of more than one nation state at a time, and the practice often revealed a narrow specialization in the rise of Prussia. But the Statists had characteristically applied comparative methods applied at a single instant in history.

In 1798 the Scottish economist Sir John Sinclair (1754-1835) published the following comment on Statistics in Germany:

> I found that in Germany they were engaged in a species of political inquiry, to which they had given the name of Statistics; and though I apply a different idea to that word, for by statistical meant in Germany an inquiry for the purpose of ascertaining the political strength of a country, or questions respecting matters of state, whereas the idea I annex to the term is an inquiry into the state of a country for the purpose of ascertaining the *quantum* of happiness enjoyed by its inhabitants, and the means of its future improvement; yet as I thought that a new word might attract more public attention, I resolved on adopting it, and I hope that it is now completely naturalised and incorporated with our language.[3]

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[2]'In dem leidenschaftlichen Streit, der zwischen Herder und Schlözer über Wesen und Behandlung der Geschichte geführt wurde, war Herder durch Weite und Freiheit des Blicks unstreitig der Überlegene; aber das Ziel, die Erhebung der Geschichte aus ödem Kleinkram zur Geschichte der bald fortschreitenden, bald entartenden Menschheit, war in beiden dasselbe.' Hermann Hettner, *Geschichte der deutschen Literatur im achtzehnten Jahrhundert*, 2 volumes, Berlin and Weimar, 1979 (first published 1870), vol. 2, p. 575.

[3]Sir John Sinclair, *The Statistical Account of Scotland*, 21 volumes, Edinburgh, 1798, vol. 20, p. xiii.

August Ludwig Schlözer (1735-1809) referred to this passage and rebutted it in 1804.[4] Appointed in 1769 to the chair of *Staatsgelehrsamkeit* at the University of Göttingen, a university which had strong ties with Great Britain, Schlözer had taken up economic and social history early in his career. Later in life he was often to contrast this favourably with an older sort of historiography which he deprecated as the work of '"anno domini" men, who merely recorded battles, changes of occupancy of thrones, and the lives of rulers, but not, however, how a people had come by its legal and financial constitution, its hereditary ruling classes, its poverty and so on.'[5] It was precisely a curiosity to discover the conditions under which the generality of the people were obliged to live that marked Schlözer out from his contemporaries. He wrote, for instance, that 'Statistics and despots cannot bear one another. There are countless weaknesses in the way a country is run that are the fault of the authorities, but by means of Statistics they can be exposed to the public. Statistics can be used to keep a check on rulers, and indeed Statistics can be a government's accuser, not something that a despot relishes. In the findings of Statistics he has a register of his sins.'[6]

But Statistics in Schlözer's sense did not survive the turn of the century. The immediate cause of its disappearance was public policy in states which feared the spread of revolutionary ideas from France. Schlözer was no supporter of revolution, but events in France were closely monitored and analyzed in his political journal the *Staatsanzeigen*. Towards the end of his life Schlözer was worried about the future of Statistics. He feared that gains he and his colleagues had made were about to be wiped out in a return to older methods and outlook. He wrote: 'Furthermore - and I'm talking about 1762, not 1800 - in those days it was those who did historical research, critics, even those who just collected variant data, who were the "notables" in the scholarly historical community. Those who merely embellished the historical narrative were left in the background, and they never dreamed that one day their grandchildren would usurp completely the good name and honour of historical thinkers.'[7] There is unpublished evidence in the Schlözer Archive in Göttingen that Schlözer sensed with foreboding the rise of *Historismus* after his death.

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[4]August Ludwig Schlözer, *Theorie der Statistik nebst Ideen über das Studium der Politik überhaupt*, Göttingen, 1804, p. 16f.

[5]'... Anno Domini Männer, die nur Schlachten, Thronveränderungen, und Biographien der Herrscher, verzeichneten, nicht aber, wie ein Volk zu seiner Justiz- und Finanzverfassung, zu seinen Erbpatriziaten und seiner Armut u. s. w. gekommen sei.' *August Ludwig Schlözer, Staatsgelahrtheit nach ihren Hauptteilen, im Auszug und Zusammenhang*, Göttingen, 1793, *Vorrede*, p. vi.

[6]'Statistik und Despotismus vertragen sich nicht zusammen. Unzählige Gebrechen des Landes sind Fehler der Staatsverwaltung: die Statistik zeigt sie an, kontrolliert dadurch die Regierung, wird gar ihr Ankläger: das nimmt der Despot ungnädig, der in solchen Angaben sein Sündenregister liest.' *Statistik*, p. 51.

[7]'Ferner, ich spreche, man merke wohl, vom Jahre 1762, nicht von 1800. Damals waren die Geschichtsforscher, die Kritiker, selbst die Variantensammler, die *notables* im historischen Volke, und hatten das Wort: die Geschichtsstaffierer standen im Hintergrunde, und träumten nicht, daß ihre Enkel den Namen und die Ehre historischer Denker ausschließlich usurpieren würden.' *August Ludwig Schlözers öffentliches und Privatleben, von ihm selbst geschrieben*, Göttingen, 1802, p. 52.

Statistics had a major international, comparative component which was rare in *Historismus*. Bielfeld said in 1767 that Statistics was properly concerned with the political arrangements of all contemporary states of the known world.[8] He claimed that Gottfried Achenwall (1719-1772), Schlözer's senior colleague at the University of Göttingen, had been the first scholar ever to write a systematic account of the subject. Achenwall stated his concern to distinguish what had developed historically, from the institutions which human society had designed at any instant in time. He wrote: 'It can assumed that there are certain practices in the political domain which can only be explained historically and not in relation to philosophy.'[9] That distinction is the theoretical basis of Schlözer's method too. The way to differentiate between the two phenomena, he argued, was to combine a chronological approach with a synchronical one, and it is the use of this second axis of analysis that distinguishes Statistics from *Historismus*. Schlözer wrote: 'Take several centuries' worth of history and cut it up into suitable periods. Next, pick out the interesting features of a state in each period and see how they relate statistically. You would end up with as many statistics as historical periods.'[10] Schlözer proposed to construe the 'statistic' of Leipzig, for example, by comparing its most recent performance with those of comparable trading centres 'Hamburg, the two Frankfurts, Amsterdam and London'. It is through comparison, he wrote, that 'the most significant component of the actual statistic of Leipzig' is elicited. Schlözer then proceeds backwards ten years at a time, eliciting a 'statistic' on each occasion, and looking for a periodicity in his results: 'figures now rising, now falling, growth, stationary points and decay of trade in Leipzig'.[11] But Schlözer never actually carries out this plan. A century later the historian Robert von Mohl noted that such was often the outcome of Schlözer's essays, commenting: 'He jumps straight into events, then moves backwards and forwards, refers to earlier studies of his own or of others and breaks off his analysis at a point where no satisfying conclusion can be reached. In this way one might deliver useful contributions but not works of national importance.'[12] Mohl goes on to

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[8] *Les premiers traits de l'érudition universelle ou analyse abregée de toutes les sciences, des beaux-arts et des belles-lettres*, par M. le Baron de Bielfeld, 3 vols., Leiden, 1767, vol. 3, p. 246.

[9] '... also kann man auch, ohne Widerspruch der Vernunft, sich gedenken, daß es in der Politik Regeln gäbe, die wenigstens noch zur Zeit nicht philosophisch, sondern bloß historisch erwiesen sind.' G. Achenwall, *Die Staatsklugheit nach ihren ersten Grundsätzen*, Göttingen, 1761, *Vorrede*, Proposition 26.

[10] 'Zerschnitte man nun, wie natürlich, eine durch mehrere Jahrhunderte fortlaufende Geschichte in schickliche Perioden, und höbe dann aus jeder bloß die Staatsmerkwürdigkeiten in engerer statistischer Bedeutung heraus; so würde dies so viele einzelne (alte) Statistiken geben, als Perioden angenommen sind.' August Ludwig Schlözer, *Theorie der Statistik*, Göttingen, 1804, p. 92f.

[11] '... bald steigende, bald fallende Zahlen, Anwachsen, Stillstand und Sinken der Leipziger Handlung ...' August Ludwig Schlözer, *Kleine Chronik von Leipzig. Erster Teil, bis zum Jahre 1466*, Leipzig, 1776, p. 8f.

[12] 'Er fällt mitten in die Begebenheiten hinein, geht dann rückwärts und vorwärts, verweist auf frühere, eigene oder fremde Arbeiten, und bricht auch wohl da ab, wo kein befriedigender Abschluß ist. Auf solche Weise aber liefert man wohl brauchbare Beiträge, aber keine Nationalwerke.' Robert von Mohl, *Die Geschichte und Literatur der Staatswissenschaften*, 3 volumes,

explain these deficencies in terms of a lack on Schlözer's part of a 'passionate predilection for his fatherland' and of a 'deeper grasp of the religious'.[13]

I argue that the underlying cause is that the theoretical basis for combining the synchronical and chronological approaches is not well founded.

It is possible that Sinclair's characterization of Statistics in Germany may be more relevant to Prussia and the kind of statistical compilation done there by Johann Peter Süßmilch, than to the academic and political activities of Schlözer. Neither Sinclair's nor Süßmilch's statistical works contain as much political economy as Schlözer's. As linear, by which I mean non-interactive, compilations, neither man's works could or did differentiate as subtly between different kinds of factor in the life of the state as had Sir William Petty (1623-1687) in his *Political Arithmetic*, published in 1690. The greatness of Petty's work lies in his attempt to show that arithmetical quantities in an economy had relative rather than absolute values. Tired of identifying mere equivalences of scale between the economies of France and England, for example, he attempted to show: 'That a small country and few people, by its situation, trade, and policy, may be equivalent in wealth and strength to a far greater people and territory'.[14] Petty applies arithmetic to the static cross-section of the economy and does not treat its history. His work is a fine example of the judicious use of induction tempered by deduction. He defines a model and then applies it to the actuality. Then he compares his results with his model. But, most importantly, he regards phenomena as mutually dependent variables of a unitary system. It is not a matter of comparing phenomena between countries simply in terms of their scale. Petty concludes with a plea for unity and tolerance:

> May not the three kingdoms be united into one, and equally represented in Parliament? Might not the several species of the King's subjects be equally mixt in their habitations? Might not jurisdictions, and pretences of power, be determined and ascertained? Might not the taxes be equally applotted, and directly applied to their ultimate use? Might not dissenters in religion be indulged, they paying a competent force to keep the public peace?[15]

Achenwall and Schlözer were more like Petty than Sinclair or Süßmilch, whose approach was pragmatic and more concerned with scale than with differentiation. Against Schlözer's requirement that Statistics should serve all the people, the Prussian state seemed to link Statistics with politics. In 'Concerning the spectre of secrecy in the Prussian states', an anonymous contribution of 1777 to one of Schlözer's statistical journals, we read: 'There is nothing new in the observation that, of all officially published material, statistical intelligence on the Prussian provinces is the rarest. But, in recent times, that cannot be said of

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Erlangen, 1855-1858, vol. 2, p. 448.

[13]'Ebenfalls nicht gerade loben, allein aus der ganzen Zeitrichtung und Umgebung erklären kann man, wenn es Schlözer offenbar an zwei Eigenschaften eines vollkommen tadellosen Schriftstellers über Zeitereignisse fehlt, nämlich an einer leidenschaftlichen Vorliebe für das Vaterland und an einer tieferen Auffassung des Religiösen.' Ibid., p. 446.

[14]Sir William Petty, *Political Arithmetic*, London, 1690, p. 1.

[15]Ibid., p. 95.

statistical commentary from the private sector. There is surely no state which has at its disposal such accurate intelligence on what is going on within itself as the Prussian.'[16] Schlözer noted that the population of Prussia had risen threefold from 69,872 in 1715 to 198,490 in 1775 ('I know of no similar occurrence in the whole of world history,' he wrote.)[17] In 1781 he noted that Prussia had recently licensed an 'open-heartedness' towards political thinkers.[18] Prussia also caught the eye of Adam Smith who commented: 'The accumulated treasures of the prince have, in former times, afforded a much greater and more lasting resource. In the present times, if you except the King of Prussia, to accumulate treasure seems to be no part of the policy of European princes.'[19] I note in passing that Adam Smith was characterized in Germany as a practitioner of 'Staatsweisheit', rather than of *Statistik*.[20] The use of 'Staatsweisheit' rather than Statistics may reflect the fact that Smith's approach was more inductive than the Statists'. Smith's purpose was to show how wealth could be created, while the Statists strove to develop methods of data collection and analysis.

Many terms found in eighteenth century German scholarly works to denote the study of politics and economics are no longer current. Examples are 'Staatsgelehrsamkeit', 'Staatsweisheit', 'Staatsgelahrtheit' or 'Staatsklugheit', each subtly different, and their multiplicity reflects a unique effort in Germany to develop the study of political economy. 'Politik' carried a more general meaning of 'political discussion' too, with 'Politiker' often employed by Achenwall and Schlözer to refer to private citizens who were interested in participating in public political discussion. In *Historismus* 'Politik' meant the actions of statesmen and politicians. Schlözer was praised by his pupil Friedrich Christian Schlosser, historian, for trying to introduce yet another kind of study which he called *Metapolitik*.[21] Statistics was the most mature form of the generality of scholarly effort in Germany, a country according to Adam

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[16]'Daß die statistischen Nachrichten von den preußischen sämtlichen Provinzen öffentlich gedruckt die raresten sind, ist ganz richtig: indessen sind es schriftlich gewiß nicht. Es ist zuverlässig kein Staat, der von dem, was in ihm vorgeht, so genaue Nachrichten hat wie der preußische.' 'Über den Geist der Verschwiegenheit in den preußischen Staaten' (Königsberg, 6. Jänner 1777) in *August Ludwig Schlözers Briefwechsel meist historischen und politischen Inhalts*, Göttingen, 1780, Teil ii, Heft vi, p. 11.

[17]Ibid., p. 12.

[18]Ibid., p. 120.

[19]Adam Smith, *An Inquiry into the Nature and Causes of the Wealth of Nations*, 2 volumes, Oxford, 1976 (first published 1776), vol. 1, p. 441.

[20]See, for instance, *Die neue Staatsweisheit oder Auszug aus Adam Smiths Untersuchung über die Natur und die Ursachen des Nationalreichtums, mit praktischen Bemerkungen von F. von Cölln*, Berlin, 1812.

[21]'Dieser geistreiche Schriftsteller hat ... mit einer Art Sehergabe den Begriff der Gesellschaftswissenschaften so ziemlich aufgefaßt und in einigen Hauptlinien unter dem Namen 'Metapolitik' gezeichnet. Allein es war dies ein vorübergehendes Meteor, ohne alle weiteren Folgen, unverstanden von allen. Ja man darf behaupten, unverstanden von Schlözer selbst ...' Friedrich Christoph Schlosser, *Geschichte des achtzehnten Jahrhunderts und des neunzehnten bis zum Sturz des französischen Kaiserreichs*, 7 volumes, Heidelberg, 1836-1848, vol. 2, p. 572.

Smith whose most eminent men of letters were professors in universities. Achenwall wrote of a struggle for acceptance of the academic study of political matters. In particular, he remembered that 'there were objections to the teaching of the subject at universities on the grounds that the absence of access on the part of cloistered academics to state secrets would inevitably mean that lectures would be shallow and useless.'[22] But Schlözer was constantly getting into trouble for publishing sensitive copy. The Bishop of Speyer, for instance, made an official complaint to the Reichstag about Schlözer's efforts to obtain simple statistical data about his territory and government. Schlözer upset the government of Bavaria in 1783 by publishing the official documents of a judicial murder in Amberg. He caused a furore at the Prussian court in 1791 by publishing a leaked copy of Frederick the Great's will.[23] This was after Schlözer had turned down an invitation to join the Prussian civil service.[24] Schlosser recalled numerous scrapes that Schlözer had had with the authorities, and he conjectured that Schlözer was protected only by the good offices of George III in London.[25] 'But it finally happened, after Schlözer had received warning after warning, that Hannover's ambassador to the Reichstag gave an express official undertaking that what the obscurantists called Schlözer's spree would be called to a halt.'[26] In 1794 Schlözer was forbidden to print any further editions of the *Staatsanzeigen*.

Although the work of Achenwall and Schlözer was largely ignored in the nineteenth century, traces of what they initiated can be seen in much later disputes over historical method in the German academic community. For instance, in a skirmish of a full-blown 'Methodenstreit' between Friedrich Meinecke of the University of Berlin and Karl Lamprecht of the University of Leipzig in 1896 about the future course of historical studies in Germany, it emerges that Lamprecht believes that a 'new' trend is appearing, in which social and economic systems are being studied more than the doings and sufferings of notable statesmen and politicians. Lamprecht wrote: 'In my opinion, it is the more recent initiatives which are introducing the notion of causality to historical study by seeking out the connections between human actions, by treating the generic life of humanity

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[22]'Man hat gegen den Vortrag dieser Wissenschaft auf Universitäten Einwürfe gemacht, als wäre solche wegen der Menge ihrer Materien voller Verwirrung, wegen der beständigen Veränderungen voller Ungewißheit, und wegen der darinnen enthaltenen Staatsgeheimnisse vor den Augen der Schulgelehrten verborgen, folglich dergleichen Vorlesungen seicht und unbrauchbar.' Gottfried Achenwall, *Abriß der neuesten Staatswissenschaften der vornehmsten europäischen Reiche und Republiken zum Gebrauch in seinen akademischen Vorlesungen*, Göttingen, 1749, p. 33.

[23]See Schlözer's *Staatsanzeigen*, No. 64, 1791.

[24]See Theodor Zermelo, *August Ludwig Schlözer, ein Publicist im alten Reich*, Berlin, 1875, p. 28f.

[25]Friedrich Christian Schlosser, *Geschichte des achtzehnten Jahrhunderts und des neunzehnten bis zum Sturz des französischen Kaiserreichs*, op. cit., vol. 1, p. 316.

[26]'Es kam endlich dahin, daß, nachdem Schlözer eine Erinnerung nach der andern erhalten hatte, der hannöverische Gesandte auf dem Reichstage die ausdrückliche Versicherung gab, daß das, was die Obskuranten Schlözers Unfug nannten, abgestellt werden solle.' Ibid., p. 317.

and not just the deeds of eminent men.'[27] Lamprecht treated his senior colleagues in much the same vein as Schlözer had his 'anno domini men'. Indeed, Lamprecht was bold enough to attack Leopold von Ranke, writing: 'It is precisely the irrational that drives history according to him.' Meinecke countered that 'freedom' was a better characterization of this element of Ranke's historical outlook, to which Lamprecht retorted that Meinecke's 'freedom' meant no more in effect than the operation of random chance ('Willkür'). Against this Meinecke, defending his own *Historismus*, put the rhetorical question: 'Can the method of statistical mass observation of phenomena, however valuable a contribution it might make in certain circumstances, ever do justice to the workings of this a priori x-factor in the mass phenomena?'[28]

The Statists were notable for their desire to promote good causes. The logic of Historist method, on the other hand, courted the danger of indifference to whether an historical development was good or evil. Most of the Historists were as liberal-minded as any of the Statists, but their historical outlook and methods, with their emphasis on the external self-interest of states ('Primat der Außenpolitik') and the outcomes of international conflict, and with their dependence on time and consequent susceptibility to moral relativity, prevented such a disposition manifesting itself in their academic publications. I argue therefore a 'primacy of synchronicity' in the testing of moral judgements, the alternative being to be obliged to see good and evil as cyclical phenomena.

Schlözer often distanced himself from the kind of search for a wider meaning or purpose in history to which Schiller referred in his inaugural lecture at the University of Jena in 1789. There Schiller speaks of one who brings 'a rational purpose into the movement of the world and a teleological principle into world history.'[29] But Schlözer wrote that improvement could only come about by enacting the appropriate changes in social organization. Progress, he said, did not occur as the result of the revelation of an unfolding purpose. 'Poland,' he wrote, 'a state of some twelve million human beings and other considerable resources, has gone under in our own times purely and simply because of the kind of political

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[27]'Vielmehr meine ich, daß die jüngeren Richtungen überhaupt erst die Kausalität rein in die Geschichte einführen, indem sie für denjenigen Teil der Personengeschichte - Personengeschichte ist alle Geschichte -, in dem es sich um das generische Leben der Menschheit, nicht um eminente Taten handelt, kausale Zusammenhänge nachweisen.' Karl Lamprecht, 'Zum Unterschied der älteren und jüngeren Richtungen der Geschichtswissenschaft', *Historische Zeitschrift*, Neue Folge 41, 1896, p. 258.

[28]'Kann nun die Methode der statistischen Massenbetrachtung, - so Wertvolles sie innerhalb gewisser Schranken auch leistet, - den Wirkungen dieses apriorischen X in den Massen je gerecht werden?' Friedrich Meinecke, 'Erwiderung auf Lamprecht', *Historische Zeitschrift*, Neue Folge 41, 1896, p. 266.

[29]'.. er bringt einen vernünftigen Zweck in den Gang der Welt und ein teleologisches Prinzip in die *Weltgeschichte*.' Friedrich Schiller, 'Was heißt und zu welchem Ende studiert man Universalgeschichte?', *Sämtliche Werke*, ed. G. Fricke and H. G. Göpfert, 5 vols., Munich, 1980, vol. 4, p. 764.

constitution it had.'[30] Again, he argued in favour of the freedom of the press and literacy in general as practical agents of social improvement: 'A writer is an unsolicited, unwaged servant of civil society, a volunteer adviser to the nation. It was through the refinement of the written word that the British developed their law of contract, and that will be the means by which, God willing, in fifty years time there will no longer be any serfs in Germany. It is through the efforts of writers that we have managed to curb the use of torture; writers have brought it about that an honest German woman can grow old without the fear of being burned as a witch.'[31]

The synchronic or simultaneous approach to history allows the treatment of many variables without having to take on the problem of accounting for the probabilistic mutual dependence of events, a problem which both Bayes and the Statists addressed and from which both subsequently withdrew. Being thus able to take a broad view, it was easier for thinkers like Petty, Smith and Schlözer, who chose an exclusively synchronic approach, to characterize invariants in terms of a spectrum of good causes. As in a physical experiment, the Historists tried in effect to minimize the effects of time on a complex system, by reducing the system to a finite number of variables, but in so doing they were compelled to take a narrow view of human needs and aspirations, and to specialize. No Historist could match the range of interests of Petty, Smith, Achenwall or Schlözer. There is no doubt that the eighteenth-century Statists revealed more about the conditions of life generally in their own time than the Historists could or would in theirs. The Historists tended to measure history along a single axis defined by the rise of Prussia. While they were not following Prussia's rising curve, historians like Schlosser, Robert von Mohl, Ranke or Droysen specialized in the 'higher' planes of action of leading politicians and statesmen. Droysen was famously induced to abandon his study of classical civilization in favour of the history of Prussia. Although Meinecke was later to develop an interest in the interactions of national and cosmopolitan political ideas, his early career was conducted along conventional lines, and his first major work was an extensive biography of the reforming General von Boyen of the era of the Wars of Liberation. The biography shows Meinecke at pains to apply a very considerable intellectual originality and imagination to rather limited and austere material. Boyen was evidently not a figure of large enough intellectual interest to serve as a vehicle for what was to become the mature Meinecke's preoccupation with the relation between German *Historismus* and wider and older patterns of European Enlightenment thought. In fact, it is debatable whether Meinecke ever succeeded in demonstrating any real connection between these traditions. Indeed, it is possible to see in Meinecke's very efforts to reach out to the wider European traditions a sense of the fundamental flaws of *Historismus*.

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[30] 'Polen, ein Staat von zwölf Millionen Menschen und anderen großen Grundkräften, ging in unseren Tagen unter, - einzig und allein seiner Regierungsform wegen.' *Theorie der Statistik*, p. 39.

[31] 'Ein Schriftsteller ist ein unberufener, unbesoldeter Diener der bürgerlichen Gesellschaft, ein *volontaire* von Ratgeber der Nation ... So haben die Briten ihre *contracts* erhalten, und so wird, (durch die Schriftsteller) will's Gott, in fünfzig Jahren kein Leibeigener mehr in Deutschland sein. Schriftsteller haben die Einschränkung der Folter veranlaßt; Schriftsteller haben es dahin gebracht, daß ein ehrliches deutsches Weib mit Ehren und ohne Furcht, als Hexe verbrannt zu werden, alt werden kann.' August Ludwig Schlözer, *Briefe nach Eichstädt. Zur Verteidigung der Publizität überhaupt und der Schlözerischen Staatsanzeigen insonderheit*, Frankfurt and Eichstädt, 1785, p. 10.

Meinecke may have yearned to break out of a narrow specialization in the history of the rise of Prussia. Schlözer, on the other hand, was attacked posthumously by Historists for his international outlook. Robert von Mohl, for example, criticized Schlözer for tending to a 'flat and broad cosmopolitanism'.[32] Schlosser accused Schlözer of being biased in favour of England as well as being a 'friend of Russia'.[33] It is noted that attacks on Schlözer went on long after his death. That suggests that his influence was considerable. Perhaps his later detractors had an underlying envy for his methods and achievements. There seems to be a feeling among them that there was more to be said about Schlözer than that he was unaware of the clashes of great nation states.

I argue that it was actually his underlying approach that fascinated later commentators, who were however insufficiently conscious of the nature of their own methods to be able to appreciate others.

I conclude by generalizing the 'uncertainty principle' to the historical sciences. The infeasibility of invertible probability made the prediction of future events on the basis of past ones uncertain. The synchronic analysis of Statistics revealed the variables of interest to the historian. But there is a large number of variables in a socio-economic system, and Bayes, admittedly in the absence of his final word on the subject, showed that it was theoretically impossible, and Schlözer showed that it was practically impossible to describe adequately how an entire system varied with time. Therefore later historians who were interested in time-dependency had to specialize in a finite number of variables. But by concentrating on a small number of variables the Historists ran the risk of indifference to the advent of new variables and ended up paradoxically with a selection of factors which was in fact arbitrary, and thus their method was highly heuristic. Statists like Schlözer had at least tried to combine the two approaches, but the Historists pursued only one. In practice, the one approach can only be taken at the expense of the other at any instant. Precision of one is gained at the expense of precision in the other.

Therefore, in the words of Heisenberg, the two approaches cannot be pursued with arbitrary precision at the same time. One can follow a quantum of history through time, but in order to do it, one must conduct an experiment in which one has a large number of points of reference at one's disposal. On the other hand, one can locate a single quantum at an instant in relation to all the other quanta, but one cannot do both at the same time.

26 September 1995

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[32]Robert von Mohl, *Die Geschichte und Literatur der Staatswissenschaften*, 3 volumes, Erlangen, 1855-1858, vol. 2, p. 446.

[33]Schlosser, op. cit., vol. 3, p. 316.

Draft 1 (7/2/96) of a paper by Geoffrey Sweet to be read before The Institute for Advanced Studies in the Humanities, Edinburgh, on 14 February 1996:

The name 'Statistics' has been associated with a range of different disciplines in both arts and sciences over the past three centuries. In general, Statistics is the study of complex phenomena, functions of many variables. At first, its object was human society. Later, it was concerned with nature; and later still, with the sophisticated artifices of technology. Recently, however, the beginnings of attempts to weave these three strands together can be discerned. Common to all three has been, and still is, the problem of explaining how and why complex systems change with time. I shall conjecture this evening that we are none the wiser today about this than were the pioneers of Statistics in the eighteenth century.

With my mind's eye I imagine Statistics first emerging as a 'horizontal' kind of inquiry after several centuries in which a more 'vertical' perception of a God-world-man continuum had been customary. I suppose that the socio-economic had played a comparatively minor role in a less complex world before the Enlightenment. Two of the earliest examples of a new 'horizontal', secular approach to man's doings and sufferings were Pufendorf's *Constitution of the German Empire* and William Petty's *Political Arithmetic*, both of the seventeenth century. But it was left to the eighteenth century to begin to consider society as it actually was. This evening I'd like to consider the success and failure of recurrent attempts to apply Mathematics to the study of society, its economics and its history. In particular, I shall conjecture that the methods of modern Mathematical Statistics arose most immediately from the perception of the infinite complexity of nature and from the actuality of complex technological systems. But I conjecture too that that perception of complexity and interactivity may have arisen originally from an enhanced awareness of man's social life and its intricate mechanisms. Roughly speaking, the notion of society as a differentiable continuum was a child of the eighteenth century, while the appreciation of the complexities of nature and technology was one of the nineteenth. The methods of Mathematical Statistics have been employed more recently in the analysis of the state. I argue that modern Mathematical Statistics differs from classical Mathematics in having a more liberal axiomatic basis. I conjecture that the pioneers of Statistics in the eighteenth century were unwilling to accept some of the axioms that form the basis of modern Mathematical Statistics, and that it is for that reason that their efforts to promote Statistics and especially to harness Statistics and Probability did not generate a lasting theoretical basis.

In my paper 'History and the principle of uncertainty', which I wrote for the Institute for Advanced Studies in the Humanities last summer, and which I read before the Eighteenth Century Studies Group at University of Exeter, I verified a claim that Statistics was a branch of learning which flourished first in Germany in the eighteenth century. I found that in Germany in those days the word 'Statistics' referred to the academic activity of generating the outlook and methods by which the elements of society may be defined and their interactions described. I paid especial attention to August Ludwig Schlözer (1735-1809) of what was then the new University of Göttingen. He was a noted historian who explored both the theoretical basis and practice of Statistics.

Synchronous analysis, the comparison of phenomena at an instant, was characteristic of Statistics in Germany in the eighteenth century, once it had settled in the academic curriculum. By then the German Statists had abandoned the study of social and economic phenomena as functions of time, even though they continued to assert Statistics to be a part of historical study. At first, the Statists had tried to elicit periodicities from historical events, but a theoretical basis for doing so proved to be elusive. Incidentally, neither Adam Smith nor before him Sir William Petty tried to, and they both had in common with the German Statists the method of synchronous differentiation of data. In particular, the Germans acknowledged a debt to Sir William Petty, who in his *Political Arithmetic*, published in 1690, suggested a differential method that went beyond simple binary comparisons of scale. An example of a study with inconsiderable differentiation is *The Statistical Account of Scotland*, instigated by Sir John Sinclair, and published first here in Edinburgh in 1798. This work attracted interest in Germany. Given the nature of this valuable compilation, an exercise in data collection and collation which was more concerned with magnitude than with interaction, it is perhaps not surprising that Sinclair misconstrued the Statistics practised in Germany, insofar as he alleged it to be an instrument for 'ascertaining the political strength of a country', rather than for 'ascertaining the *quantum* of

happiness enjoyed by its inhabitants.'[1] I treated Schlözer's rebuttal of Sinclair's characterization in my paper last summer. Schlözer's idea of Statistics was not to provide governments with data, but rather to provide data about governments.

I contrasted eighteenth century Statistics with the outlook and methods of German historiography afterwards in the nineteenth and early twentieth centuries. *Historismus*, the characteristically German historiography of that later era, attended to a comparatively small number of variables in accounting for the motions of societies. The Statists, on the other hand, believed that they had discovered that the key to unlocking the secrets of economics consisted in assuming a system of massive differentiablity. However, the practical difficulties of setting limits to the differentiation hampered their efforts to lay a straightforward theoretical basis for explaining the causes and means by which a complex system changed with time. But Historists, on the other hand, characteristically studied developments over time, an activity which was facilitated, as I have argued elsewhere, by their selection of comparatively few factors. A well-known one, for instance, flowed from a hypothesis that nation states could increase their wealth only if their people's ingenuity were supplemented by military might. Often, the recent rise of Brandenburg Prussia was studied in order to test the contribution that military prowess made to that nation state's evident political strength. In the event, allowing such a hypothesis as well as certain other axioms which licensed the primacy of foreign policy over domestic policy, the Historists construed a model according to which the relative prosperity of nation states could be explained in terms of one nation state's being in the ascendancy in the context of another's, or others', decline. Clearly, in a practical study, a more or less generous time frame had to be allocated in order to differentiate between ascent and descent. But the nation state itself was an integral concept which the eighteenth century Statists' methods would not have allowed, and, in general, the Historists accepted axioms more readily in laying the theoretical basis of their work than had the Statists.

I considered last summer the theoretical limits of explaining the behaviour of systems of more than a few variables over time. I drew upon Mathematical theory and practice for guidance as I conjectured that the Statists were obliged to accept the primacy of synchronicity in their analysis because of the large number of variables that they supposed constituted a social system. The eighteenth century German historians have often been said, especially by Historists, not to have had much of an historical sense. But I argued that they were impelled to make their analysis independent of time by the mathematical impossibility of analysing systems of such complexity as functions of time, in other words, of integrating systems of such intricate differentiability over time. The Historists, on the other hand, often proceeded from an idea that historical change was itself the energy of social interaction, and that social organization was a second order effect. It is well known among historians that there was what is called the 'crisis of *Historismus*' in German historiography after the First World War. The crisis occurred largely because the outcome of the war was not what Historists had expected. The world in the 1920s had not become what they had been predicted. In particular, the state of the world did not seem any longer to be explicable in terms of a Germano- or Prussocentric view of history.

In another paper, '*Historismus* and experimental philosophy', published last year in the *Modern Language Review*, I compared the German historical outlook and methods of the nineteenth and early twentieth centuries with those of the Quantum Physics and Relativity, which grew up at the same time, and I found allegations in both branches of learning that certain exceptions from the principles of scholarly inquiry needed to be made in order to explain certain phenomena. In the case of the Quantum Physics, this entailed modifying or neglecting indeed the notion of continuity for integration, which had been essential to the theoretical basis of classical Mathematics. For instance, if a physical effect could be demonstrated which could not be explained in terms of classical Mathematical principles, as was sometimes argued, then the remedy might be to devise a new kind of Mathematics. Likewise, with *Historismus*, it was sometimes found convenient to apply a notion of the 'organic' rise and fall of institutions in an investigation where it was

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[1]Sir John Sinclair, *The Statistical Account of Scotland*, 21 volumes, Edinburgh, 1798, vol. 20, p. xiii.

found that elements which in theory should not have been present simultaneously, nevertheless were found to be present simultaneously. Time was very often suggested as the key to such problems. It might be conjectured, for instance, that an incongruent phenomenon was in a flux of decay, would diminish in due course anyway, and could therefore be neglected. In the physical experiment, there was a danger of confusing the terms of the experiment and its inevitable physical limits with the natural phenomenon under investigation, to the point where it was perhaps difficult to tell which, of natural events or human intervention, had played the greater part. Much could be done to encourage a certain kind of world-view by these methods, but it was not always obvious whether human knowledge of nature had been advanced by them. On the one hand, credence of uncertain longevity could be lent to certain political ideas, while, on the other, such a practical device as the atom bomb could be envisaged and built. But it was not obvious in either case how much progress had been made with human understanding of the world or of nature.

The Statists of the eighteenth century German Enlightenment proceeded from the premise that it was good for society to collect data about itself. The task of Statistics was to distil information from the data collected. The nineteenth century Historists often sneered at their eighteenth century academic colleagues for the abstract character of much of what they produced. In so doing they echoed the criticisms of some of the Statists' own eighteenth century contemporaries, who argued, among other things, that academic people were not situated to discover the motives of rulers and administrators. The Historists were less inclined to question the facts of the state, just as they were less inclined to examine the theoretical tenets of their work. But the Statists were interested in the way in which information can be gleaned from data. They were especially interested in finding out the nature of change in society from one instant to the next. Their concerns to discover thus the invariants of human ambition and conduct are contiguous with the efforts of their contemporary Immanuel Kant to isolate the conditions and terms on which an inquiry into ethical precepts and laws before experience could be conducted. The Historists, on the other hand, proceeded from the point which the Statists had in many ways tried to avoid, namely the fact of partiality and prejudice in the observation and analysis of phenomena. The Historists perceived little invariance in historical development powered by the vital and the organic, as they saw it, and they argued with great rigour and brilliance from that premise. The crisis of *Historismus* of the 1920s, however, suggested that such premises had been false. The Statists, on the other hand, wanted to clarify and exemplify the premises. One way of helping that process was to find out as much as possible about how the body politic functioned from day to day. In my paper last summer I gave examples of a burgeoning investigative journalism in Germany on the eve of the French Revolution. Schlözer was notably spurred by a frustration with what he regarded as the calculated obscurity in which much public policy was made and the incompetence and malice of many who were charged to implement it. Hence Schlözer's characterization as a trouble-maker by representatives of some German states, a characterization echoed in the nineteenth century by the many historians who regarded their first duties in the light of loyalty to their employer, the state. Schlözer's investigative journalism was disliked and combatted in many official institutions in Germany and Austria. His political and statistical journals were suppressed in many places, but he himself enjoyed the freedom from censorship enjoyed at the University of Göttingen, a freedom which was approved of and protected in London.

I propose now to consider the relations between Mathematics and Mathematical Statistics. The Mathematics which children have customarily learned at school is orientated towards the physical sciences and is characterized by the analysis of systems consisting of a few variables. Usually these are three coordinates of space and one of time. However, since the onset of physical experiments on a grander scale through the nineteenth century, and a concomitant rise of technology, with its complexity of energy-saving mechanical or, nowadays, electronic systems, in which it is axiomatic that more should be gained through the system for human ease and advancement than is lost in terms of energy, there has been a recognition that many variables need to be accounted for in order to describe the behaviour of a system. The social and economic systems studied by the eighteenth century Statists were of a complexity which, could it be described at all in Mathematical terms, had no analogy in the Mathematics of the day. Indeed, the relations between economic and social study and Mathematics are uncertain to this day. However, modern Mathematical Statistics is a discipline which is designed to bridge the divide.

The success of failure with which the outlook and methods of modern Mathematical Statistics are applied to actual problems is very much to do with the nature of the problem itself. In the case of social and economic phenomena, these are today, as always, of a complexity that many believe defies Mathematical treatment. However, their complexity does not always defy modern Mathematical Statistical treatment. For instance, a problem may be shown not to have a solution at all or not a unique solution, if considered Mathematically. But, if considered in terms of Mathematical Statistics, on the other hand, it might be shown to have multiple solutions or 'a solution in the limit of repeated experiments', in other words one which might emerge at the end of a process of trial and error. Many problems are said to be amenable only to 'statistical' methods because of some assumptions that have had to be made about the phenomenon under investigation. An example would be a phenomenon assumed to vary in time only with respect to one or two, or a very few, at least, variables upon which it depends. Strictly speaking, such a problem would be described in classical Mathematical terms as a system undergoing a partial differentiation with respect to one variable, all other variables being held constant. In theory, certainly, it is possible to conceive of systems which, when undergoing change, can be assumed not to be changing in terms of other factors of which it consists. However, the idea that time itself is a differentiator of a system which changes in respect of a finite number of respects, with all others held constant, is a fairly recent one, characteristically of our present century, and it is partly licensed by modern Mathematical Statistics, and I shall try to exemplify that later. But without such a theory it is difficult to see how one can distinguish cause and effect, a relationship which, if it reveals itself at all, can surely only reveal itself over time. This was a weakness that the eighteenth century German Statists acknowledged in their synchronical approach. But their synchronical approach enabled them to isolate and describe many more facets of society than they might otherwise have been able to do. The later Historists, on the other hand, were much more confident that they could distinguish between cause and effect, but they characterized the system undergoing change in such a narrow way, that they failed to notice a multitude of significant factors and determinants.

Schlözer once proposed a method of dividing the history of a society up into cross-sections, and then comparing the different cross-sections one with another. He hoped that this approach might give clues as to how to relate a state of affairs at one instant with that at another. He hoped to find patterns and periodicities emerging over time. But in the end he could not find a theoretical justification for thinking that such patterns or periodicities existed, nor was he able to identify any non-trivial example of such an effect.

In my paper of last summer I referred to the English Probabilist Thomas Bayes (1702-1763) and a celebrated paper entitled 'An essay towards solving the doctrine of chances', attributed to him and communicated after his death to the Royal Society by one Richard Price and read there on the 23rd of December 1763. In a modern introduction to the text, reprinted in the journal *Biometrika* in 1958, it is noted that:

Thomas Bayes's famous Essay is so often referred to in current statistical literature, but so rarely studied because of difficulty of access, that the Editors have felt justified in reprinting it ...[2]

Bayes is often cited in Statistics and Engineering textbooks and papers as the author of a notion glossed variously as 'inverse probability', 'inductive probability' or '*a priori* probability'. What is variously referred to as 'Bayes' formula', 'Bayes' rule' or 'Bayes' theorem' is an expression for the result of experiments conducted in the past and a consideration of the probability of the same result occurring in a future experiment. Price reported that Bayes himself had posed the problem in the following words:

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[2]G. A. Barnard, 'Thomas Bayes - a biographical note', *Biometrika*, December 1958, volume 45, p. 293.

Given the number of times in which an unknown event has happened and failed: *Required* the chance that the probability of its happening in a single trial lies somewhere between any two degrees of probability that can be named.[3]

In what is known today as 'Bayes' rule', 'Bayes' formula' or 'Bayes' theorem',[4] the probabilistic variable appears as a denominator on the right hand side of the equation. Prediction of the value of it requires proving the feasibility of taking the reciprocal of the probability and rearranging the equation so that the probability appears as a result on the left hand side of the equation rather than as a condition on the right hand side. However, a probability must be treated as an uncertainty in Mathematics, and there is no theoretical basis for conceiving of its reciprocal. Although today's exponents of 'Bayesian' methods implicitly assume the existence of the reciprocals of such uncertainties, a reading of the unfinished paper attributed to Bayes reveals that its author was unable to prove it. It is unfortunate that Price did not see fit to communicate Bayes' notes in their entirety, especially those parts of them which might have given us a clue as to his actual conclusions. Price explained that he had omitted such notes because they would have taken up too much room, and because they were not as cogent as he might have hoped them to be:

The two last rules in this essay are given without the deductions of them. I have chosen to do this because these deductions, taking up a good deal of room, would swell the essay too much; and also because these rules, though of considerable use, do not answer the purpose for which they are given as perfectly as could be wished.(*Biometrika* 45, 1058, p. 297)

I argue that Bayes' service to Mathematics was rather to show that Algebra is a discipline which imposes severe limits on the treatment of uncertain events. It is important that Bayes discovered that Algebra did not permit the removal of an uncertainty from one side of an equation to the other by inversion.

There is no evidence that Bayes himself wished his paper to be published, and I quote Sir Ronald Fisher, an eminent Statistician, who wrote in 1959:

Bayes' introduction of an expression representing probability *a priori* thus contained an arbitrary element, and it was doubtless some consciousness of this that led to his hesitation in putting his work forward. ...

A more important question, however, is whether in scientific research, and especially in the interpretation of experiments, there is cogent reason for inserting a corresponding expression representing probabilities *a priori*. This practical question cannot be answered peremptorily, or in general, for certainly cases can be found, or constructed, in which valid probabilities *a priori* exist, and can be deduced from the data. More frequently, however, and especially when the probabilities of contrasted scientific theories are in question, a candid examination of the data at the disposal of the scientist shows that nothing of the kind can be claimed.[5]

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[3]'"An essay towards solving a problem in the doctrine of chances" by the late Rev. Mr. Bayes, F. R. S., communicated by Mr. Price, in a letter to John Canton, A. M., F. R. S., read 23 December 1763', *Biometrika* 45, 1058, p. 298.

[4]See, for instance, K. S. Shanmugan and A. M. Breipohl, *Random Signals: Detection, Estimation and Data Analysis*, New York, 1988, p. 19.

[5]R. A. Fisher, *Statistical Methods and Scientific Inference*, Second Edition, Edinburgh and London, 1959, p. 17.

Price reveals an anxiety about a low esteem granted to probabilistic Mathematics in his own time. However, referring to the Mathematician de Moivre (1667-1754) , French-born, but British by adoption, who was interested in probability, and who in his *Laws of Chance* (1716) had developed rules for estimating relative probabilities on the basis of completed trials, he says:

Mr. De Moivre calls the problem he has thus solved, the hardest that can be proposed on the subject of chance. His solution he has applied to a very important purpose, and thereby shewn that those are much mistaken who have insinuated that the Doctrine of
Chances in mathematics is of trivial consequence, and cannot have a place in any serious enquiry.

However, it was one thing to distil probability ratios from events that had already occurred, but quite another to estimate the probability of future occurrences. If anything, Bayes showed that what is today referred to as 'invertible probability', 'inductive probability' or '*a priori* probability' was theoretically uncertain. Indeed, a scepticism about the calculability of time seems to be characteristic of eighteenth century thought in general. While the Statists had hoped to gain something for our understanding of the invariants in human aspirations and needs, so the Historists tended to license the dependence of mankind upon the workings of time. Again, the Historists were able to demonstrate only a limited system changing with time, while the Statists had concluded that it was necessary to concentrate on the simultaneous, 'static' disposition of phenomena at an instant. One of the well-known weaknesses of *Historismus*, one acknowledged by Friedrich Meinecke and Ernst Troeltsch, distinguished exponents of the genre in the twentieth century, was that the notion of the dependence of society, its outlook and methods of organisation, upon time could easily come to license a moral relativism.

I will now illustrate the difference between Thomas Bayes' treatment of 'invertible probability' and a common twentieth century view of the matter by considering the outlook and methods of the celebrated twentieth century Mathematical Statistician A. N. Kolmogorov (1903-1987). In 1933 Kolmogorov caused the publication of his own short treatise entitled *Basic Concepts of Probability Theory*.[6] The Preface of this work begins with the words:

The purpose of the present volume is an axiomatic justification of probability theory.[7]

Kolmogorov expands upon the role of axioms in his analysis as follows:

Probability theory as a mathematical discipline is to be and can be axiomatized in exactly the same way as Geometry or Algebra. That means that once the names of the objects requiring to be investigated and their basic connections as well as the axioms, which these basic connections have to obey, have been given (set out), the whole further exposition is to be based exclusively upon these axioms and may take no account of whatever might happen to be the concrete meaning of those objects and connections.[8]

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[6]A. Kolmogoroff, *Grundbegriffe der Wahrscheinlichkeitsrechnung*, in *Ergebnisse der Mathematik und ihrer Grenzgebiete*, herausgegeben von der Schriftleitung des 'Zentralblatt für Mathematik', zweiter Band, 1933

[7]'Zweck des vorliegenden Heftes ist eine axiomatische Begründung der Wahrscheinlichkeitsrechnung.' (*Grundbegriffe*, III).

[8]' Die Wahrscheinlichkeitstheorie als mathematische Disziplin soll und kann genau in demselben Sinne axiomatisiert werden wie die Geometrie oder die Algebra. Das bedeutet, daß, nachdem die Namen der zu untersuchenden Gegenstände und ihrer Grundbeziehungen sowie die Axiome, denen diese Grundbeziehungen zu gehorchen haben, angegeben sind, die ganze weitere Darstellung sich ausschließlich auf diese Axiome gründen soll und keine Rücksicht auf die jeweilige konkrete Bedeutung dieser Gegenstände und Beziehungen nehmen darf.' (*Grundbegriffe*, p. 1)

These strictures provide a good example of a self-assured quality of Kolmogorov's writing. He does not expand upon the relations between the axioms and the objects of study, whether, for instance, the one emerges from the other in the form of a process of inductive or deductive reasoning; nor does he refer to any consensual sources of the axioms.

Kolmogorov wishes to make the basic concepts of Statistics compatible with what he calls 'the general conceptual configurations of modern Mathematics.'[9] But he does not define or explain what he means by 'modern Mathematics'. It is probable, however, that he is referring to the 'new' kind of Mathematics envisaged by some Quantum Physicists. Kolmogorov mentions that the concepts of Mathematical Statistics 'had been considered to be quite peculiar until recently', suggesting perhaps that Probability was still, two hundred years on, enjoying only an uncertain Mathematical respectability.

Kolmogorov distinguishes between the treatment of a finite and an infinite number of occurrences. For infinite numbers of occurrences he prescribes what he calls 'essentially new principles.'[10] Kolmogorov does not state the theoretical basis of such new principles, but their effect was evidently to make infinite quantities manageable in terms that seemed familiar in the context of classical Mathematics.

Kolmogorov defines an infinite series of occurrences as one the average value of which is zero and he defines the probability of a single occurrence in the limit as the *n*th occurrence tends to infinity likewise as zero. This definition of the infinite Kolmogorov terms the 'steadiness axiom' ('Stetigkeitsaxiom', *Grundbegriffe*, p. 13) and it is the sixth and final axiom which Kolmogorov lays down. He declares that his 'system of axioms' is 'free of contradiction' ('widerspruchsfrei'), while conceding that it is incomplete ('unvollständig'). However, it can be argued that since the theoretical basis of each axiom is not adumbrated by Kolmogorov, it is difficult to define the phenomena to which the six axioms might apply. A circularity is revealed in which objects suitable for analysis are defined by the methods by which they are analysed. The term 'axiom' is certainly an appropriate one to employ in such a context. However, mutual logical consistency is not thought to be a defining quality of axioms, and the mutual coherence of Kolmogorov's axioms is indeed uncertain.

The efficacy of Kolmogorov's method depends upon the tractability of the matter to which it is applied. His treatment of the infinite, for example, is limited to such phenomena of which it is known that their occurrence is independent of any other possible occurrence. Kolmogorov seems not to share Bayes' unease with the appropriateness of Algebra for describing properties of the infinite. Indeed, it is consistent with Kolmogorov's general axiomatic approach that he states a theorem which he says Bayes formulated, a theorem which invites the licensing of the inversion of an infinity. Unfortunately, it cannot be discovered where Kolmogorov found the theorem of Bayes that he quotes (*Grundbegriffe*, p. 7). It is perhaps odd that Kolmogorov appears to rule out the possibility on inverting probability in his own axioms, especially the sixth one, yet appears to be licensing exactly that in citing what he calls 'Bayes'theorem'.

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[9] 'Der leitende Gedanke des Verfassers war dabei, die Grundbegriffe der Wahrscheinlichkeitsrechnung, welche noch unlängst für ganz eigenartig galten, natürlicherweise in die Reihe der allgemeinen Begriffsbildungen der modernen Mathematik einzuordnen.' (*Grundbegriffe*, p. III)

[10] 'Wir nennen elementare Wahrscheinlichkeitsrechnung denjenigen Teil der Wahrscheinlichkeitsrechnung, in welchem Wahrscheinlichkeiten von nur endlich vielen zufälligen Ereignissen vorkommen. Die Sätze, die hier gewonnen werden, werden natürlich angewandt auch auf Fragen, die mit unendlich vielen zufälligen Ereignissen verbunden sind, allerdings braucht man bei der Behandlung dieser letzteren Fragen auch wesentlich neue Prinzipien.' (*Grundbegriffe*, p. 1)

I summarize. The problem which the eighteenth century German Statists came up against was the treatment of multi-variate systems and the ways they changed over time. They acknowledged that it was ultimately impossible to define all the elements of which society and societies consisted, but they began their study on the assumption that all elements were mutually dependent. The later Historists, on the other hand, effectively freed a number of factors in the life of the collectivity from mutual interdependence, but the limits of this strategy were laid bare in the 'crisis of *Historismus*'. Likewise, it seemed convenient to many Quantum Physicists and Relativists to regard certain effects or occurrences as independent of others, except that some, notably Niels Bohr, whose Correspondence Principle was designed to stress continuity with classical Mathematics and to recall the need to promote the analytical self-evidence of experimental results, were worried about the implications of such an assumption. Kolmogorov's 'axioms' are of a piece with the modified Mathematics propounded in some Quantum Physics. In particular, his definition of unknown, or future, as the case might be, events as those which are 'statistically independent' might not have seemed peculiar to those practitioners of Quantum Physics who defined quanta as discontinuous events that could not be accounted for in terms of the 'traditional' Mathematical requirement for continuity in integration. Kolmogorov considered the possible role of axioms in helping over the difficulties that Bayes had faced. Bayes had stuck to the letter of the Algebraic law, and his results showed that Algebra offered no more to aid the prediction of future events than could common sense.

The problem of how to treat massively multi-variate problems is is much to the fore today. Much work is done to construct models in order to predict economic activity, for instance. The complexities of the global economy seem to defy Mathematical methods, but Statistical strategies for solving such complex problems, such as the Genetic Algorithm, in which the effects of a single factor such as a parasite are simulated in order to explain biodiversity, or Simulated Annealing, where the states of systems are examined in the presence of difference temperatures and different rates of change of temperatures, do not yield better than optimal or sub-optimal solutions. In both cases, the hope is cherished that a single parameter, or at least a very small number of parameters, can be identified which will account for the diverse manifestations of a complex system. The notion of neural nets includes the possibility of employing more than a few parameters. But what is uncertain about neural nets is how independent the mechanism of the analysis can be of the behaviour of the phenomenon under scrutiny. Again, a multiplicity of solutions may be the result.

I conjecture that 'Statistics' came originally from an awareness of the complexity of socio-economic life. In the eighteenth century, society was widely taken to be the paradigm of a much-to-be differentiated phenomenon. In the eighteenth century one can identify early efforts to construe information about the state. The first steps were taken by some universities in Germany. At the same time, inconclusive measures were taken to apply mathematical methods to socio-economic problems. This initiative foundered upon a theoretical uncertainty about the prediction of future events. From the late eighteenth century onwards statistical inquiry became more an activity of the state, rather than of academic inquiry. However, developments in science and technology spawned new Mathematical methods for treating multi-variate problems. However, the most effective Mathematical methods for treating multi-variate systems are found in the realm of what are called 'stationary' systems, in other words, complex man-made systems which which were designed to be calculable in terms of time from the beginning. But neither nature nor society are like that.

I venture a definition of 'Mathematical Statistics' as the licensing of discontinuity for the purpose of integration. This sets Mathematical Statistics at odds with classical Mathematics, where variables must be specified and systems known to be continuous before integration is attempted. Furthermore, whereas the methods of classical Mathematics can scarely treat problems of more than about five variables, Statistics' essential purpose is to tackle problems of very many variables, the selection of which is uncertain. Where classical Mathematics would require certainty in the selection of variables, Statistics allows noise, and probability is founded on the notion of random effects. The task of Probability is to distinguish what is random from what is essential in a system, but it cannot license a definition of the system in the first place. But Mathematical Statistics suffers from theoretical difficulties in distinguishing noise from uncertainty and

method from event. I believe that in the eighteenth century both Schlözer and his colleagues, and Thomas Bayes had a grasp of these difficulties, resisted the employment of axioms on principle, and committed themselves to allowing the limits of theory to be set by practical experience. The twentieth century, by contrast, does rather tackle the problem from the opposite direction, running the risk, as I have suggested, of ending up defining matter in terms of theory, or, in the case of Kolmogorov, problems in terms of axioms. Likewise, the twentieth century has spawned a number of popular political creeds which have proceeded from uncertain assumptions about human beings, whose executors found in the event that only a finite number of human beings were susceptible to such assumptions. Schlözer and his colleagues took a more deductive approach. In particular, Kant busied himself with the whole idea of moral precepts before experience, and one of the valuable lessons from his work was that the pursuit of the 'best', as opposed to an 'optimal' course of action at least had positive educational implications. It remains to be seen whether Mathematics can been applied to economic or social problems with greater effect than was attempted in the eighteenth century. It is possible that the eighteenth century will continue for some time to have the latest, if not indeed the last, word on the usefulness of classical Mathematics in socio-economic study.